Int. J. Heat Mass Transfer. Vol. 18, pp. 469–472. Pergamon Press 1975. Printed in Great Britain

SHORTER COMMUNICATIONS

EVALUATION OF MICROLAYER CONTRIBUTION TO BUBBLE GROWTH IN NUCLEATE POOL BOILING USING A NEW BUBBLE GROWTH MODEL

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(*Received* 26 *April* 1974)

NOMENCLATURE

R_1, bubble base radius [m];
H, total bubble height [m];
R, radius of the sphere of which bubble is a segment [m];
R_{EQ}, equivalent radius of bubble [m];
V_b, volume of the bubble [m^3];
t, dimensionless time ($=\tau/\tau_d$);
B_1, bubble base growth constant [m];
n_1, m_1, bubble base growth indices;
B, bubble height growth constant [m];
n, bubble height growth index;
$R_{1,\max}$, maximum bubble base radius [m];
v, fluid velocity [m/s];
y, distance normal to heating surface [m];
C_p, specific heat [J/kg K];
k, thermal conductivity [W/m K];
h_{fg}, latent heat of vaporisation [J/kg];
T_{sat}, temperature of saturation [K];
T_1, initial temperature of heating surface [K];
τ, time [s];
τ_d, departure time [s];
γ, kinematic viscosity of liquid [m^2/s];
δ_0, initial microlayer thickness [m];
δ, instantaneous microlayer thickness [m];
ρ, density [kg/m^3];
α, thermal diffusivity [m^2/s];
$\Gamma(x)$, gamma function;
${}_1F_1(\alpha, \beta, z)$, confluent hypergeometric series.

Subscripts

S, heater (solid);
L, liquid;
V, vapour.

BUBBLE GROWTH MODEL

RECENT theories of bubble growth in nucleate pool boiling postulate that a major portion of heat transfer to the bubble occurs by conduction through a liquid microlayer formed on the heated surface. It is therefore important that the base contact area of the bubble, through which heat is transferred, is accurately accounted for. For this purpose, a new bubble growth model as shown in Fig. 1 is considered. In this model the shape of a growing bubble is represented by a spherical segment of which both the radius of the base in contact with the heating surface, R_1, and the vertical height above the base, H, vary with time (τ). The boundary conditions on $R_1(\tau)$ and $H(\tau)$

$$R_1(0) = R_1(\tau_d) = 0$$
$$H(0) = 0$$
$$H(\tau_d) = 2R$$

where R is the radius of the sphere of which the bubble is a segment and τ_d is the departure time, are satisfied if R_1 and H have the forms

$$R_1 = B_1 t^{n_1}(1-t^{m_1}) \quad (1)$$

$$H = Bt^n \quad (2)$$

where $t = \tau/\tau_d$ and B_1, n_1, m_1, B and n are empirical constants.

Equations (1) and (2) are empirical in nature but appear to represent the physical bubble shape reasonably well. Measurements of R_1 from bubble photographs (after 10-fold magnification) of several investigators [1–4] as well as those taken by the authors during boiling of water over a polished brass surface, using an apparatus similar to that of Han and Griffith [1], showed that $n_1 \approx 1{\cdot}0$ and $m_1 \approx 0{\cdot}2$. Comparing equation (1) with

$$R_1 = C_1 t^{1/2} \quad (3)$$

during the interval $0{\cdot}05 < t < 0{\cdot}4$, we find that

$$t^{1/2}/9{\cdot}9 < t - t^{1\cdot2} < t^{1/2}/8{\cdot}1.$$

So, either equation (1) or (3) may be made to fit the same bubble by suitable choice of B_1 and C_1 (e.g. $B_1 \approx 9C_1$). Beyond the above range, the two equations differ appreciably. It is necessary to use the best fit for R_1 because this is the only shape factor which enters the analysis of microlayer evaporation. Figure 2 shows that plots of R_1 vs $(t - t^{1\cdot2})$ give nearly straight lines. This means that equation (1) may be considered to represent experimental data reasonably well.

GENERAL EXPRESSION FOR MICROLAYER THICKNESS

Assuming that during the initial rapid growth period (period of increasing R_1) the bubble remains nearly hemispherical, the horizontal velocity v_{r_1} induced in the liquid in the vicinity of the heating surface at a radial distance r_1 from the bubble centre is related to the wall velocity v_{R_1} of the bubble having radius R_1, by

$$v_{r_1} = \frac{R_1^2}{r_1^2} v_{R_1}. \quad (4)$$

Using equations (1) and (4)

$$v_{r_1} = \left(\frac{B_1^3}{r_1^2 \tau_d}\right) \sum_{i=1}^{4} N_i t^{S_i - 1} \quad (5)$$

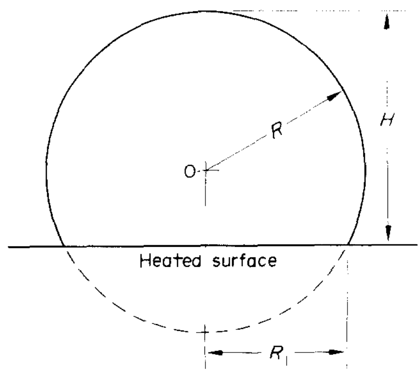

FIG. 1. Bubble growth model.

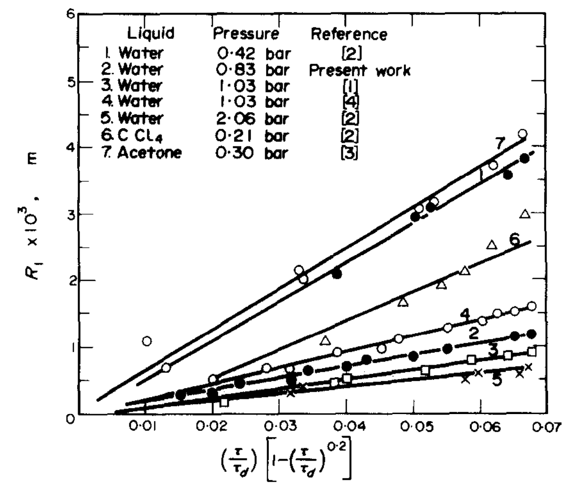

FIG. 2. R_1 vs its time function.

where $N_1 = n_1$; $N_2 = -(3n_1+m_1)$; $N_3 = 3n_1+2m_1$; $N_4 = -(n_1+m_1)$; $S_1 = 3n_1$; $S_2 = 3n_1+m_1$; $S_3 = 3n_1+2m_1$ and $S_4 = 3(n_1+m_1)$. If the heated surface is now assumed to move in a direction opposite to fluid motion with velocity v_{r_1} and v represents the resulting velocity profile in the liquid the microlayer thickness δ_0 [5] under the bubble edge where $v_{r_1} = v_{R_1}$ will be

$$\delta_0 = \left|\frac{1}{v_{r_1}}\int_0^\infty v\,\mathrm{d}y\right|_{r_1=R_1} \tag{6}$$

where y is the distance normal to the heated surface. The velocity $v(y,\tau)$ may be obtained from the solution of simplified Navier-Stokes equation

$$\frac{\partial v}{\partial \tau} = \gamma_L \frac{\partial^2 v}{\partial y^2}; \quad y > 0; \quad \tau > 0 \tag{7}$$

with $v(y,0) = 0$; $v(0,\tau) = v_{r_1}$ where γ_L = kinematic viscosity of the liquid.

Using Laplace–Carson transformation equation (7) becomes

$$\frac{\mathrm{d}^2L(v)}{\mathrm{d}y^2} - q^2L(v) = 0; \quad q = \sqrt{(p/\gamma_1)}; \quad \gamma_1 = \gamma_L\tau_d \tag{8}$$

and

$$L(v)\Big|_{y=0} = \left(\frac{B_1^3}{r_1^2\tau_d}\right)\sum_{i=1}^4 N_i p^{1-S_i}\Gamma(S_i). \tag{8a}$$

The solution of equation (8) is

$$L(v) = \left(\frac{B_1^3}{r_1^2\tau_d}\right)\sum_{i=1}^4 N_i p^{1-S_i}\Gamma(S_i)\,\mathrm{e}^{-qy}$$

which on inversion and simplification [6] becomes

$$v(y,\tau) = \left(\frac{B_1^3}{r_1^2\tau_d}\right)\sum_{i=1}^4 N_i t^{S_i-1} \times \left[\mathrm{e}^{-X}\,{}_1F_1(S_i-1/2;1/2;X) - 2\Gamma(S_i)X^{1/2} \times \mathrm{e}^{-X}\,{}_1F_1(S_i;3/2;X)/\Gamma(S_i-1/2)\right] \tag{9}$$

where

$$X = \frac{y^2}{4\gamma_1 t}; \quad {}_1F_1(\alpha,\beta,z) = \frac{\Gamma(\beta)}{\Gamma(\alpha)}\sum_{k=0}^{\infty}\frac{\Gamma(\alpha+k)z^k}{\Gamma(\beta+k)k!}.$$

Using equation (9) and integral formulae from reference [7]

$$\int_0^\infty v\,\mathrm{d}y = \left(\frac{B_1^3}{r_1^2\tau_d}\right)\sqrt{(\gamma_1 t)}\sum_{i=1}^4 N_i J(S_i)t^{S_i-1} \tag{10}$$

where

$$J(S_i) = \Gamma(S_i)/\Gamma(S_i+1/2).$$

From equations (6) and (10) the local thickness of the microlayer formed under the bubble edge is given by

$$\delta_0 = \sqrt{(\gamma_1 t)}\left[\sum_{i=1}^4 N_i J(S_i)t^{S_i-1}\Big/\left(\sum_{i=1}^4 N_i t^{S_i-1}\right)\right]. \tag{11}$$

Van Ouwerkerk [8] has shown that the boundary-layer separation does not occur if $R_1 = C_1 t^{1/2}$. It is found that the velocity v_{r_1} given by equation (5) is higher than that for van Ouwerkerk's model (with $B_1 = 9C_1$, $n_1 = 1{\cdot}0$, $m_1 = 0{\cdot}2$) during the time in which $0{\cdot}09 < R_1/R_{1,\max} < 0{\cdot}92$. Since the period during which $R_1 > 0{\cdot}92R_{1,\max}$ is relatively unimportant for vapour formation, it is reasonable to assume that microlayer separation will not appreciably affect the bubble growth.

HEAT TRANSFER TO BUBBLE

The temperature profiles $T_L(y,\tau)$ in the microlayer and $T_s(y,\tau)$ in the solid are given by the following heat-conduction equations (y being the distance normal to the solid–liquid interface and positive when measured from interface into the solid).

$$\frac{\partial T_L}{\partial \tau} = \alpha_L\frac{\partial^2 T_L}{\partial y^2}; \quad -\delta < y < 0 \tag{12}$$

$$\frac{\partial T_s}{\partial \tau} = \alpha_s\frac{\partial^2 T_s}{\partial y^2}; \quad y > 0 \tag{13}$$

with

$$k_L\left(\frac{\partial T_L}{\partial y}\right)_{y=0} = k_s\left(\frac{\partial T_s}{\partial y}\right)_{y=0} \tag{14}$$

$$T_L(0,\tau) = T_s(0,\tau) \tag{15}$$

$$T_L(y,0) = T_s(y,0) = T_s(\infty,\tau) = T_1 \tag{16}$$

$$T_L(-\delta,\tau) \approx T_{\mathrm{sat}}. \tag{17}$$

The solution of equations (12)–(17) gives [9]

$$T_L = T_1 - (T_1 - T_{sat}) \sum_{n=0}^{\infty} \eta^n \times \left[\operatorname{erfc} \frac{(2n+1)\delta + y}{2\sqrt{(\alpha_L \tau)}} - \eta \operatorname{erfc} \frac{(2n+1)\delta - y}{2\sqrt{(\alpha_L \tau)}} \right] \quad (18)$$

where

$$\eta = (\sigma - 1)/(\sigma + 1); \quad \sigma = [(k_s \rho_s Cp_s)/(k_L \rho_L Cp_L)]^{1/2};$$

α = thermal diffusivity; k = thermal conductivity; ρ = density; Cp = specific heat; and the suffixes s and L denote solid and liquid respectively. Also T_1 and T_{sat} are initial temperature of the heating surface and saturation temperature respectively.

For boiling of saturated or nearly saturated bulk liquid, the heat transfer to the bubble in terms of microlayer thickness decrease is given by (h_{fg} = enthalpy of vaporization)

$$\rho_L h_{fg} \frac{d\delta}{d\tau} = -k_L \left(\frac{\partial T_L}{\partial y} \right)_{y=-\delta}. \quad (19)$$

Using equations (18) and (19), we obtain

$$\frac{d\delta}{d\tau} = -[k_L(T_1 - T_{sat})/[\rho_L h_{fg} \sqrt{(\pi \alpha_L \tau)}]] \times \left[1 + 2 \sum_{n=1}^{\infty} \eta^n e^{-n^2\delta^2/(\alpha_L \tau)} \right]. \quad (20)$$

It has been shown by van Ouwerkerk that the ratio σ has negligible effect on the bubble growth. A very simple case then emerges by assuming this ratio to be unity (i.e. $\eta = 0$). This amounts to the assumption of infinitely thick microlayer and equation (20) reduces to

$$\frac{d\delta}{d\tau} = -[k_L(T_1 - T_{sat})/[\rho_L h_{fg} \sqrt{(\pi \alpha_L \tau)}]]. \quad (21)$$

If τ_g is the time instant of microlayer formation and τ_{gd} the instant when the microlayer is no longer present at a base radius R', then

$$\int_{\tau_{gd}}^{\tau} \frac{d\delta}{d\tau} d\tau = 0; \quad \tau > \tau_{gd}.$$

The volume V_b of bubble formed as a result of microlayer evaporation may then be written as

$$V_b(\tau) = -(\rho_L/\rho_V) \int_0^{R_1} 2\pi R' \int_{\tau_g}^{\tau} \frac{d\delta}{d\tau} d\tau \, dR'; \quad \tau < \tau_{max} \quad (22)$$

or

$$V_b(\tau) = -(\rho_L/\rho_V) \left[\int_0^{R_1} 2\pi R' \int_{\tau_g}^{\tau} \frac{d\delta}{d\tau} d\tau \, dR' + \int_{R_1}^{R_{1,max}} 2\pi R' \int_{\tau_g}^{\tau_{gd}} \frac{d\delta}{d\tau} d\tau \, dR' \right]; \quad \tau > \tau_{max} \quad (23)$$

where τ_{max} corresponds to $R_{1,max}$ and ρ_V is the vapour density. The equivalent radius R_{EQ} of a spherical bubble having volume V_b is then given by

$$R_{EQ} = [3V_b/(4\pi)]^{1/3}. \quad (24)$$

RESULTS

The results obtained on the basis of the above theory (curves 1) have been compared with those of the previous theories (curves 2–5) as well as with experimental results in Figs. 3 and 4. Experimental values of R_{EQ} and the empirical constants B_1, n_1 and m_1 were computed from the measured values of R_1 and H.

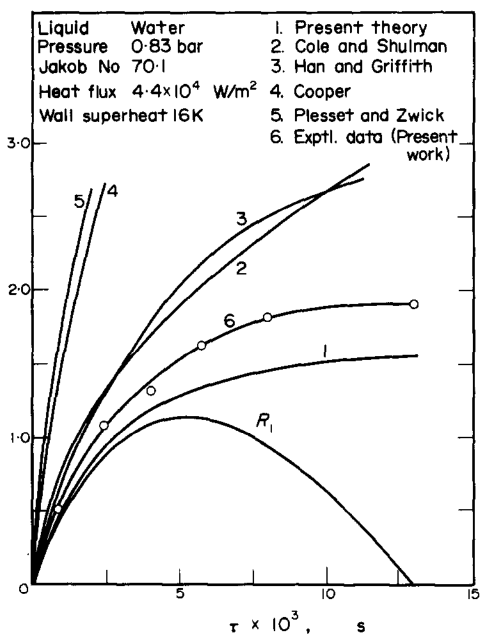

FIG. 3. Comparison of theoretical and experimental bubble growth results.

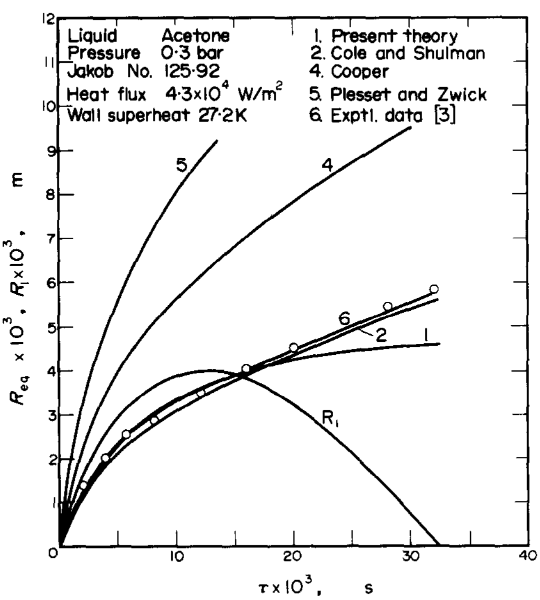

FIG. 4. Comparison of theoretical and experimental bubble growth results.

CONCLUSIONS

1. It appears more appropriate to assume that the bubble shape is represented by a spherical segment characterized by the base radius R_1 and the vertical height H.

2. A general expression has been developed for initial microlayer thickness.

3. The bubble growth curves obtained on the basis of the assumption of "infinitely thick microlayer", i.e. $k_s \rho_s Cp_s = k_L \rho_L Cp_L$ are in satisfactory agreement with experimental results.

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Int. J Heat Mass Transfer. Vol. 18, pp. 472–474. Pergamon Press 1975. Printed in Great Britain

EFFECT OF JAKOB NUMBER ON FORCES CONTROLLING BUBBLE DEPARTURE IN NUCLEATE POOL BOILING

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(*Received* 23 *July* 1974)

NOMENCLATURE

C_d, C_{do}, drag coefficients;
C_{pL}, specific heat of liquid [J/kg-K];
D, bubble diameter [m];
D_c, diameter of bubble neck in contact with heating surface at departure [m];
D_d, bubble diameter at departure [m];
F, force normal to heating surface [N];
g, acceleration due to gravity [m/s^2];
h, average heat-transfer coefficient [W/m^2-K];
h_{fg}, latent heat of vaporization of bulk liquid [J/kg];
k, thermal conductivity of liquid [W/m-K];
N_{Ja}, Jakob number ($C_{pL}\rho_L \Delta T/h_{fg}\rho_V$);
$\dot{q}$, heat flux [W/m^2];
t, time [s];
T_w, wall temperature [°K];
T_{sat}, saturation temperature [°K];
T_∞, bulk liquid temperature [°K];
ΔT, wall superheat [°K];
V_g, velocity of center of bubble, $\frac{1}{2}\frac{dD}{dt}$ [m/s];
V_n, normal velocity of bubble front, dD/dt [m/s];
V_t, terminal velocity [m/s];
δ, thickness of superheated liquid layer [m];
ρ_L, liquid density [kg/m^3];
ρ_V, vapor density [kg/m^3];
σ, coefficient of surface tension [N/m];
θ, contact angle [rad].

INTRODUCTION

IT IS well known that at low values of Jakob number the bubble growth rates and departure diameters are small and bubble departure is controlled primarily by the surface tension force, the inertia (drag and liquid inertia) forces being relatively small. At high values of Jakob number, on the other hand, the growth rates and departure diameters are large and inertia forces control bubble departure. The object of this paper is to obtain quantitatively the Jakob number ranges over which the surface tension force and inertia forces control, respectively, the process of bubble departure.

THEORETICAL ANALYSIS

Assumptions

The following assumptions have been made:

1. At the instant of departure the bubble is spherical in shape and it is attached to the heating surface by a short neck of diameter D_c and having a contact angle $\theta \approx \pi/2$.